HOMEWORK 1

2018年5月21日 20:39

MATH2010E HOMEWORK 1

Please do the following problems from "Thomas' Calculus" due to May 23, 5 pm.

Exercises 12.3: 6, 15, 28

Exercises 12.4: 34

Exercises 12.5: 6, 47, 58, 64

Exercises 12.6: 4, 10, 12, 30.

Exercises 12.3

Dot Product and Projections

In Exercises 1–8, find

a. $\mathbf{v} \cdot \mathbf{u}$, $|\mathbf{v}|$, $|\mathbf{u}|$

b. the cosine of the angle between $\mathbf{v}$ and $\mathbf{u}$

c. the scalar component of $\mathbf{u}$ in the direction of $\mathbf{v}$

d. the vector $\text{proj}_{\mathbf{v}}\,\mathbf{u}$.

1. $\mathbf{v} = 2\mathbf{i} - 4\mathbf{j} + \sqrt{5}\mathbf{k}$, $\mathbf{u} = -2\mathbf{i} + 4\mathbf{j} - \sqrt{5}\mathbf{k}$

2. $\mathbf{v} = (3/5)\mathbf{i} + (4/5)\mathbf{k}$, $\mathbf{u} = 5\mathbf{i} + 12\mathbf{j}$

3. $\mathbf{v} = 10\mathbf{i} + 11\mathbf{j} - 2\mathbf{k}$, $\mathbf{u} = 3\mathbf{j} + 4\mathbf{k}$

4. $\mathbf{v} = 2\mathbf{i} + 10\mathbf{j} - 11\mathbf{k}$, $\mathbf{u} = 2\mathbf{i} + 2\mathbf{j} + \mathbf{k}$

5. $\mathbf{v} = 5\mathbf{j} - 3\mathbf{k}$, $\mathbf{u} = \mathbf{i} + \mathbf{j} + \mathbf{k}$

6. $\mathbf{v} = -\mathbf{i} + \mathbf{j}$, $\mathbf{u} = \sqrt{2}\mathbf{i} + \sqrt{3}\mathbf{j} + 2\mathbf{k}$

6.(a) $v \cdot u = (-1, 1, 0) \cdot (\sqrt{2}, \sqrt{3}, 2)$

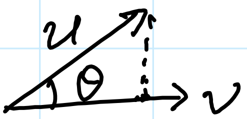
$$= -\sqrt{2} + \sqrt{3}$$

$$|v| = \sqrt{1^2 + 1^2} = \sqrt{2}$$

$$|u| = \sqrt{\sqrt{2}^2 + \sqrt{3}^2 + 2^2} = \sqrt{2+3+4} = 3.$$

(b) $$\cos\theta = \frac{v \cdot u}{|v||u|} = \frac{-\sqrt{2}+\sqrt{3}}{3\sqrt{2}}.$$

(c) scalar component $= |u| \cos\theta$



$$= (-\sqrt{2} + \sqrt{3})/\sqrt{2}$$

(d) $\operatorname{Proj}_v u = \dfrac{u \cdot v}{|v|^2} v = \dfrac{-\sqrt{2}+\sqrt{3}}{2}(-i+j)$

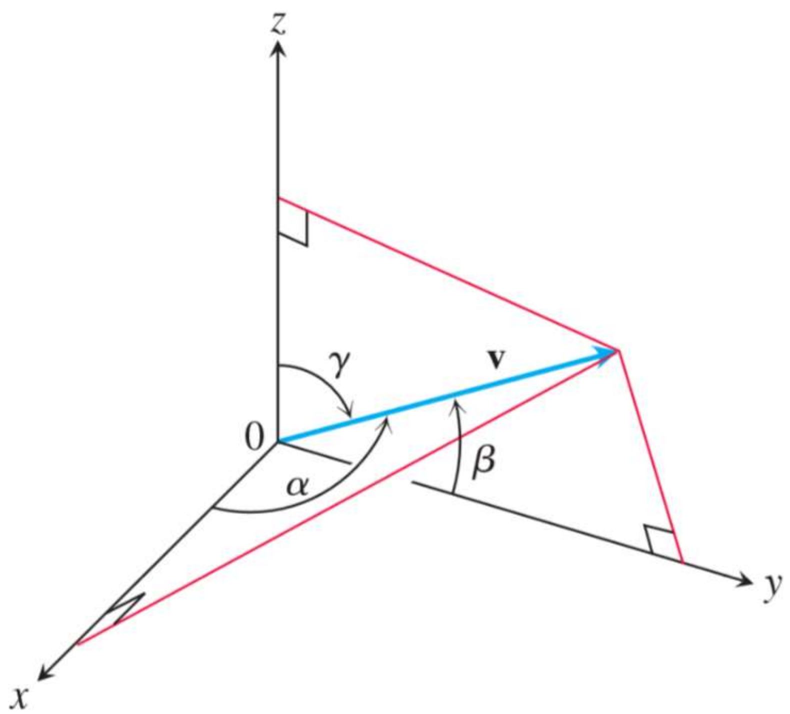
$$= \frac{\sqrt{2}-\sqrt{3}}{2} i + \frac{-\sqrt{2}+\sqrt{3}}{2} j.$$

15. Direction angles and direction cosines The *direction angles* α, β, and γ of a vector $\mathbf{v} = a\mathbf{i} + b\mathbf{j} + c\mathbf{k}$ are defined as follows:

α is the angle between $\mathbf{v}$ and the positive x-axis $(0 \le \alpha \le \pi)$

β is the angle between $\mathbf{v}$ and the positive y-axis $(0 \le \beta \le \pi)$

γ is the angle between $\mathbf{v}$ and the positive z-axis $(0 \le \gamma \le \pi)$.



a. Show that

$$\cos\alpha = \frac{a}{|\mathbf{v}|}, \qquad \cos\beta = \frac{b}{|\mathbf{v}|}, \qquad \cos\gamma = \frac{c}{|\mathbf{v}|},$$

and $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$. These cosines are called the *direction cosines* of $\mathbf{v}$.

b. Unit vectors are built from direction cosines Show that if $\mathbf{v} = a\mathbf{i} + b\mathbf{j} + c\mathbf{k}$ is a unit vector, then a, b, and c are the direction cosines of $\mathbf{v}$.

15. (a) $\cos\alpha = \dfrac{v \cdot i}{|v| \cdot |i|} = \dfrac{a}{|v|}$.

$\cos\beta = \dfrac{v \cdot j}{|v| \cdot |j|} = \dfrac{b}{|v|}$.

$\cos\gamma = \dfrac{v \cdot k}{|v| \cdot |k|} = \dfrac{c}{|v|}$

$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = \dfrac{a^2+b^2+c^2}{|v|^2} = 1$.

(b) v : unit vector $\Rightarrow |v| = 1$.

$\cos\alpha = \frac{a}{|v|} = a$. $\cos\beta = \frac{b}{|v|} = b$. $\cos\gamma = c$.

Thus a, b, c are the direction of cosines of v.

28. **Cancellation in dot products** In real-number multiplication, if $uv_1 = uv_2$ and $u \neq 0$, we can cancel the u and conclude that $v_1 = v_2$. Does the same rule hold for the dot product? That is, if $\mathbf{u} \cdot \mathbf{v}_1 = \mathbf{u} \cdot \mathbf{v}_2$ and $\mathbf{u} \neq \mathbf{0}$, can you conclude that $\mathbf{v}_1 = \mathbf{v}_2$? Give reasons for your answer.

28. No.

Consider $u = (1,0,0)$, $v_1 = (0,1,0)$, $v_2 = (0,0,1)$

$\Rightarrow u \cdot v_1 = u \cdot v_2 = 0$, but $v_1 \neq v_2$.

This is a counter-example.

Exercises 12.4

33. Cancelation in cross products If $\mathbf{u} \times \mathbf{v} = \mathbf{u} \times \mathbf{w}$ and $\mathbf{u} \neq \mathbf{0}$, then does $\mathbf{v} = \mathbf{w}$? Give reasons for your answer.

34. Double cancelation If $\mathbf{u} \neq \mathbf{0}$ and if $\mathbf{u} \times \mathbf{v} = \mathbf{u} \times \mathbf{w}$ and $\mathbf{u} \cdot \mathbf{v} = \mathbf{u} \cdot \mathbf{w}$, then does $\mathbf{v} = \mathbf{w}$? Give reasons for your answer.

34. Yes.

Known $u \times (v - w) = 0$, $u \cdot (v - w) = 0$.

Define $\tilde{u} \triangleq v - w$. $\theta \triangleq$ angle between $u, \tilde{u}$.

$0 = |u \times \tilde{u}| = |u||\tilde{u}| \sin\theta$. & $|u||\tilde{u}|\cos\theta = 0$.

$u \neq 0 \Rightarrow |\tilde{u}|\sin\theta = 0, \ |\tilde{u}|\cos\theta = 0$.

Since $\sin\theta$ and $\cos\theta$ cannot be 0 at the same time,

we have $|\tilde{u}| = 0$. $\Rightarrow v - w = 0$. $\square$

Exercises 12.5

Find parametric equations for the lines in Exercises 1–12.

1. The line through the point $P(3, -4, -1)$ parallel to the vector $\mathbf{i} + \mathbf{j} + \mathbf{k}$
2. The line through $P(1, 2, -1)$ and $Q(-1, 0, 1)$
3. The line through $P(-2, 0, 3)$ and $Q(3, 5, -2)$
4. The line through $P(1, 2, 0)$ and $Q(1, 1, -1)$
5. The line through the origin parallel to the vector $2\mathbf{j} + \mathbf{k}$
6. The line through the point $(3, -2, 1)$ parallel to the line $x = 1 + 2t, y = 2 - t, z = 3t$

6. Direction vector: $(2, -1, 3)$

$\Rightarrow$ equation: $x = 3 + 2t$

$y = -2 - t$

$z = 1 + 3t$.

Angles

Find the angles between the planes in Exercises 47 and 48.

47. $x + y = 1, \quad 2x + y - 2z = 2$

48. $5x + y - z = 10, \quad x - 2y + 3z = -1$

47. normal vectors: $n_1 = (1, 1, 0) \quad n_2 = (2, 1, -2)$.

Angle between plane = Angle between normal vectors

$$= \operatorname{Arccos} \frac{n_1 \cdot n_2}{|n_1||n_2|} = \operatorname{Arccos} \frac{3}{\sqrt{2} \cdot 3} = \operatorname{Arccos} \frac{1}{\sqrt{2}} = \frac{\pi}{4}.$$

Find parametrizations for the lines in which the planes in Exercises 57–60 intersect.

57. $x + y + z = 1, \quad x + y = 2$

58. $3x - 6y - 2z = 3, \quad 2x + y - 2z = 2$

58. Choose a common point: $(1, 0, 0)$.

Two normal vector $n_1 = (3, -6, -2)$, $n_2 = (2, 1, -2)$.

$$n_1 \times n_2 = \begin{vmatrix} i & j & k \\ 3 & -6 & -2 \\ 2 & 1 & 2 \end{vmatrix} = (-10, -10, 15) \mathbin{/\!/} (-2, -2, 3)$$

This is the direction vector of the line.

$$\Rightarrow \begin{cases} x = 1 - 2t \\ y = -2t \end{cases}$$

$$\Rightarrow \begin{cases} x = 1-2t \\ y = -2t \\ z = 3t. \end{cases}$$

64. Use the component form to generate an equation for the plane through $P_1(4, 1, 5)$ normal to $\mathbf{n}_1 = \mathbf{i} - 2\mathbf{j} + \mathbf{k}$. Then generate another equation for the same plane using the point $P_2(3, -2, 0)$ and the normal vector $\mathbf{n}_2 = -\sqrt{2}\mathbf{i} + 2\sqrt{2}\mathbf{j} - \sqrt{2}\mathbf{k}$.

64. ① $(x-4)-2(y-1)+(z-5)=0.$

② $-\sqrt{2}(x-3)+2\sqrt{2}(y+2)-\sqrt{2}z=0.$

Exercises 12.6

4. $-x^2+y^2+z^2=0.$ Cone.

Let x fixed, we get a circle. $\Rightarrow$ g.

10. $z=-4x^2-y^2.$ Parabolla.

$z<0$, fixed $\Rightarrow$ ellipse. $\Rightarrow$ f.

12. $9x^2+4y^2+2z^2=36 \Rightarrow$ ellipsoid.

12. … $\Rightarrow$ ellipsoid.

$\left(\frac{1}{3}, \frac{1}{2}, \frac{1}{\sqrt{2}}\right)$: radius $\Rightarrow$ C.

30.